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THE IRONY OF ESSENCE:
PROCLUS AND DESCARTES ON GEOMETRY

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1. *Descartes' Platonism: Between Frege and Proclus*

A direct reference to the ancient tradition is one of the main features of the birth of modern science. As starting point of his 1591 *Introduction to the Analytical Art* François Viète choses a reference to Antiquity: «In mathematics there is a certain way of seeking the truth, a way which Plato is said to have discovered, and which was called “analysis” by Theon».¹ Similarly Salviati states in the first day of Galilei's *Dialogue on the Two Chief World Systems*: «Plato himself admired the human understanding and believed it to partake of divinity simply because it understood the nature of number, I know very well; nor am I far from being of the same opinion».² Shifting the reference to Plato's successors, at the beginning of his *Harmony of the World* Kepler shows all his admiration for the Platonic tradition by stating that: «If [Proclus] had let to us his commentary to the tenth books of Euclid as well, he would both have freed our geometers from ignorance, if he had not been neglected, and relieved me totally from this toil of explaining the distinguishing features of geometrical objects».³ In doing so, the rhetorical strategy of the Moderns is to present their new science as a restitution and accomplishment of what Plato and his followers started. Inevitably, this rhetoric created the lasting prejudice that the modern

¹ Viète 1968, 320. On Viète's notion of analysis see Panza 2007.

² Galilei 1967, 11.

³ Kepler 1997, 9. On Kepler and Proclus see Claessens 2011.

understanding of the world can be read as a transformed, but loyal form of Platonism.⁴

A first difficulty faced by this kind of interpretation is that Platonism is a complicated and controversial concept. Leaving aside the open reference to the Platonic dialogues or to Plato's self-proclaimed heirs in Late Antiquity, it seems quite impossible to detect a stable core of doctrines which represent the Platonic teaching *par excellence*.⁵ In this puzzling landscape mathematics seems to ensure a safe point of departure. As the previously quoted passages show, at the beginning of modern times the reference to the Platonic tradition was particularly strong in those authors engaged in building a new understanding of mathematics. The alleged confidence in our post-Fregean gaze – with a Platonic, i.e. realistic, conception of mathematical entities – let the connection between the modern mathematical interpretation of nature and Platonism seem plausible.⁶ This made it possible to include within the tradition of Platonism the majority of authors who dealt with mathematics at the very beginning of its modern history, even in the case where the explicit reference to Platonic conceptuality is not as strong as in Kepler or Galilei. It is precisely the case of Descartes, who says in the Fifth Meditation:

The truth of these matters [mathematical ideas] is so open and so much in harmony with my nature, that on first rediscovering them it seems that I am not so much learning something new as remembering what I knew before [*ante sciebam reminisci*]; or it seems like noticing for the first time things which were long present within me although I had never turned my mental gaze on them.

⁴ This kind of interpretation is one of the grounds of the contemporary history of science, since it can be found in the groundbreaking Koyré 1933 and 1965. However, from this point of view, Koyré is a diligent student of Natorp, Cassirer and Husserl. See Natorp 1882, Cassirer 1947 and Husserl 1970.

⁵ Extremely instructive on the many possible interpretations of a “Platonic” framework in 16th century is Galluzzi 1973.

⁶ The problems of this connection are well explained by Klein 1985, Funkenstein 1986 and Ferrarin 2014.

But I think the most important consideration at this point is that I find within me countless ideas of things which even though they may not exist outside me still cannot be called nothing; for although in a sense they can be thought of at will, they are not my invention [*non tamen a me finguntur*] but have their own true and immutable natures [*suas habent veras et immutabiles naturas*]. When, for example, I imagine a triangle [*triangulum imaginor*], even if perhaps such figure does not exist, or has never existed, anywhere outside my thought, there is still a determinate nature, or essence, or form of the triangle which is immutable and eternal, and not invented by me or dependent on my mind. This is clear from the fact that various property can be demonstrated of the triangle [...]; and since these properties are ones which I know clearly recognize whether I want or not, even if I never thought of them at all when I previously imagined the triangle [*cum triangulum imaginatus sum*], it follows that they cannot have been invented by me.⁷

Our post-Fregean understanding of mathematics inevitably leads us to read these lines as an endorsement of a realistic interpretation of mathematical objects. Actually, this is the main interpretative option for many Cartesian scholars.⁸ These readings do not take into account the actual features of Cartesian mathematics. On the contrary, they read Descartes according to our contemporary notion of Platonic realism in mathematics which, *ça va sans dire*, has nothing to do with the cursory and scattered discussions on mathematics in Plato's dialogues. As I hope to show, this account is untenable if we look at the Fifth Meditation as a presentation of the ontology presupposed by Cartesian *Geometry*.

However, there is also another way to look at the problem. It consists in reading Descartes' ontology of mathematics against the background of the 16th century debate over the nature of mathematical knowledge,⁹ the so called *quaestio de certitudine mathematicarum disciplinarum*. The key issue in the debate was to establish the explicative capacity of

⁷ AT VII, 63.25-64.24; Descartes 1995, 44-45.

⁸ Gueroult 1953, Kenny 1970 and Scribano 2006 are the most relevant. See the critique of this approach in Schmaltz 1991.

⁹ The most interesting and compelling account is in Mancosu 1996.

mathematical demonstrations, and the related connection to physics and metaphysics.¹⁰ The *quaestio* is an incredibly rich attempt to recast the complex Scholastic reflection on mathematics, following the innovations which mark the peculiarity of the 16th century discussion. From the 13th to 16th century, Western mathematics faced the deepest transformation in its history with the rediscovery of the Greek geometrical and arithmetical heritage, together with the introduction of the Arabic technique of calculus called “algebra” as its driving force.¹¹ This profound change raised the need of a new understanding of the structural grounds of mathematical knowledge and practice. On this point, modern intellectuals faced the trickiest problem for everyone who reads Ancient mathematical texts: the almost complete lack of methodical and epistemological explanations.¹² Consequently, the texts where they could find indications concerning the conceptual background of Ancient Greek practices obtained a crucial role of mediation. After his publication in 1533 and Latin translation in 1560, Proclus’ *Commentary* to the First Book of Euclid’s *Elements* emerged as the main source of historical information, methodological organization of the Ancient mathematical practice, and ontological explanation of the nature of mathematical entities.¹³ Especially his critique of the Aristotelian account of mathematical objects as abstracted from the senses rapidly became one of the crucial reference in the debate over the nature of mathematical knowledge. It was used to criticize the Thomist account of mathematics as a product of a formal abstraction from the sensible objects, which was extremely widespread in the Scholastic culture.¹⁴ In other words, the attempt to substitute Aristotle’s authority with that of Proclus was the first step which gave rise to the debate over the nature of mathematics.

If this explains the presence of a Platonic conceptuality in the debate, nothing has yet been said about Descartes. He studied Euclid thanks to Clavius’ commentary which starts with a direct reference to Proclus

¹⁰ De Pace 1992 and, more recently, Higashi 2018.

¹¹ For a good introduction to this topic see Catastini, Ghione, Rashed 2016. The best books on the rise of Western algebra are still Klein 1968 and Mahoney 1973.

¹² On this point see the interesting remarks in Knorr 1993.

¹³ Cf. Helbing 2000.

¹⁴ On Aquinas’ understanding of mathematics see Maurer 1993 and Schultz 1994.

and Plato¹⁵ and he was acquainted with Kepler's texts where Proclus is often taken as a point of departure for the discussion of the ontology of mathematics.¹⁶ Nevertheless, it is impossible to state if Descartes actually read Proclus. Probably he did not. However, scholars frequently refer to some crucial affinities. This would especially be the case with Descartes' anti-Aristotelian account of mathematics, according to which mathematical objects are given to the mind without any reference to the sense-perception. Another topic, where a connection has been detected, is the role of the imagination in mathematics. In particular, Proclus is usually credited as the inventor of the so called "productive imagination" (whatever this may mean when detached from its Kantian context). This conception is usually coupled with Descartes' analysis of imagination in the *Rules for the Direction of the Mind*.¹⁷ According to such a view,¹⁸ Descartes' alleged mathematical Platonism would be the heir of the Platonic tradition in the sense that it is a renewal of Proclus' interpretation of the Euclidean *Elements* under many fundamental aspects.

2. *Mathematics is said in many ways*

Mathematics has a history. Like every other cultural history, it is a complexity of attempts, breaks, and *cul-de-sac*, with the temptation to speak of itself as a continuity. These trivial statements are usually forgotten when we try to understand the kind of conceptuality which philosophers make use of in trying to give an account of mathematical practice. Surely, mathematics and philosophy of mathematics are not

¹⁵ See Rodis-Lewis 1987 on Clavius as Descartes' source. On Clavius' mathematical epistemology cf. Claessens 2009.

¹⁶ On Kepler's influence on the formation of Cartesian thought see Gäbe 1972.

¹⁷ Especially Bouriau 2002 and Rabouin 2009.

¹⁸ Let me note that this view does not exist in the organic and unitary way I described. The relationship between Proclus and Descartes is usually seen through the lens of a specific topic like the epistemic role of the imagination in mathematics, the debate on the concept of *mathesis universalis*, Descartes' relation to the partisans of the Proclean account (especially Barozzi and Biancani) and so on. See the literature quoted in the previous footnotes for more specific references. However, the most organic and complete confrontation is Nikulin 2002.

the same. They have their own different historical traditions and not always they mutually converge. Nevertheless, this circumstance does not free us from a careful enquiry concerning the kind of mathematics which philosophers are dealing with.

Unlike the common historical *vulgata*, Ancient mathematics was perfectly aware of the universality of the methods of deduction and demonstration.¹⁹ One of its central questions was how the general dimension of the operational practice could be related to entities conceived as singular and individual. This made up the ontology of mathematical entities, its capacity to sustain the demand of generality of the practice, and our capacity to keep in touch with it, the actual *crucis* of ancient thought on the topic.²⁰ Despite the fact that these are the leading questions even in Proclus' case, his approach is different. In fact, the presupposition for an insight into the ontology of the object of mathematics is a correct understanding of its *practice*. Proclus is first and foremost interested in explaining the functioning of the working activity of the mathematician or, more narrowly, of the geometrician as Euclid presents it in the First Book of the *Elements*.

Despite the attempt to create a continuous flux of derivation, Euclidean methodology is clearly divided into two different and mutually irreducible practices. The first one requires showing that a certain property belongs to an entity, while the second one requires the execution of a multiplicity of different actions. Euclid does not provide a classification of these two procedures, but they are clearly distinct from each other by the famous ending rhetorical formulas ὅπερ ἔδει δεῖξαι (“this had to be shown”) and ὅπερ ἔδει ποιῆσαι (“this had to be done”). Following a tradition as old as Archimedes, Proclus names the first θεώρημα (“theorem”) and the second πρόβλημα (“problem”).²¹ He makes several attempts to define this couple of terms, and one of the most accurate is the following:

This second part [which follows the principles], in geometry, is divided into the working out of problems and the discovery of

¹⁹ See Acerbi 2011.

²⁰ On this topic see Klein 1968.

²¹ Cf. Mugler 1958 and Acerbi 2010.

theorems. It calls “problems” [προβλήματα] those propositions whose aim is to produce, bring into view or construct what in a sense does not exist and “theorems” [θεωρήματα] those whose purpose is to see, identify, and demonstrate that something belongs or not. Problems require us to construct a figure, or set it at a place, or apply it to another, or inscribe it in or circumscribe it about another, or fit it upon, or bring it into contact with another, and the like; theorems endeavor to grasp firmly and bind fast by demonstration the attributes and inherent properties belonging to the objects that are the subject-matter of geometry.²²

The lack of a unitary category describing the multiple kinds of “doing” in the problem is noteworthy. If the activity connected to a theorem is always a demonstration, the problem demands different actions, which are impossible to reduce to a unique practice. Thus, the theorem has a direct and univocal reference, while the problem is mainly defined by its “not being a theorem”. According to Proclus the impossibility to find a unity in the problem is the sign that it does not have a stable ontological reference. Thus, the problem depends on the theorem.

It is important to stress that such a hierarchy does not find any correspondence in the Euclidean text. The *Elements* is a masterpiece of epistemic balance and ontological reticence. However, what is clear is that the geometrician must always be respectful of the different kinds of entities adapting the methods to them.²³ However, Euclid’s silence leaves the door open to more specific interpretations of the nature of the mathematical entity. Thus, Proclus can introduce a hierarchy between the two procedures, which are valued according to their loyalty to the being of a separated and self-sustained form located in the *voûç*.²⁴

Following the book VII of Plato’s *Republic*,²⁵ Proclus strongly

²² In Eucl. 201, 3-15; Proclus 1992, 157-158 (modified translation).

²³ On this point see Lachterman 1989 and Mueller 2006.

²⁴ More generally, on the relation between Euclidean geometry and Proclus’ metaphysics see Breton 1969, Charles-Saget 1982 and Schmitz 1997.

²⁵ Cf. *Rep.* 527b4-5 [Plato 1991, 206] where we are told by Socrates that geometry «is for the sake of knowing what is always, and not at all for what is at any time coming into being and passing away». This implies that geometricians are wrong when they «speak as though they were men of action and were making all the arguments for the

separates the ontology of mathematics from the epistemological practices of the working mathematicians. Thus, the distinction between theorem and problem coincides with different ways according to which the soul can deal with the separate noetic entity. In particular, the problem is the way in which the separate and intangible entity is brought back to the conditions under which the soul is able to know. That means that the soul should be provided with the capacity of producing the visible appearance of the object. This capacity is what Proclus calls φαντασία, imagination. Actually, «it is in imagination that constructions, sectionings, superpositions, comparisons, additions, and subtractions take place».²⁶ The imagination of the soul is the place where the immutable forms receive a generation which does not belong to their essence or being. Thus, in a sense Proclus' notion of imaginative activity is productive. It has precisely the task to make visible the geometrical entity through its fictional generation. Consequently, generation and production are not acts of the imagination as such, but its contents. If we want to say that there is a production in Proclean imagination, this is a production of the figural visibility of the form and *not of the entity*. However, the possibility to fill the gap between the intangible entity and the imaginative generation requires the intervention of a different capacity which plays a crucial role in Proclus' understanding of mathematics: the διάνοια. However, in order to understand the meaning of this concept in Proclus and its difference from its original Platonic use in the *Republic*, we have to be extremely clear on the mathematical phenomenon which its employment has to explain.

As we will see later when dealing with Descartes, modern geometry is a unitary field. Its unity finds expression in the language which modern mathematics uses in order to identify kinds of objects which, in ancient thought, belong to different domains of being. This language is algebra, which was completely unknown to ancient Greek mathematicians. Basically, algebraic calculus reduces every possible enquiry concerning different entities to a matter of ratios and proportions making them the

sake of action, uttering sounds like squaring, applying, adding, and everything of this sort» (527a7-9).

²⁶ In Eucl. 78.25-79.1; Proclus 1992, 64.

unitary domain of mathematics.²⁷ On the contrary, ancient geometry can be differentiated into two, mutually irreducible elements: the image and the discourse thanks to which the epistemic power of the image can be shown.²⁸ Nevertheless, the ancient enquiry on the nature of geometry was first and foremost focused on the ontology of the entity pictured in the image. More clearly stated, the problem was mainly to understand the way of being of a plane figure or a solid discussing their ontology and our capacity to focus on them *without taking into account the need of geometry of a discursive articulation of the content of the image*. Of course, the point here is not the logical functioning of language in the demonstration (as Aristotle does in the *Analytics*, for example). Ancient geometry and, more in general, ancient mathematics is not reducible to a linguistic field. Thus, the crucial point is to understand the mutual interplay between image and language.

Proclus' argument radically assumes this division in order to articulate the relation between the knowing soul and the noetic form. Actually, the soul can have two different kinds of representation of the intelligible geometrical essence:

Geometry asks the question “What is it?” and that in two senses: it wants either the definition and notion or the actual being of the thing [τὸν λόγον ζητεῖ καὶ τὴν νόησιν, ἢ τὴν οὐσίαν αὐτὴν τοῦ ὑποκειμένου]. I mean, for example, when it asks: “What is the homoeomeric line?” it wishes to find the definition [τὸν ὄρον] of such a line, namely, “the homoeomeric line is a line all of whose parts fit upon each other” or to grasp the actual species [ἀντὰ τὰ εἶδη] of homoeomeric line, that is, the straight line, the circular line, or the cylindrical helix.²⁹

²⁷ One of the best examples of the partisanship of modern mathematics for a unitary field is its repulsion for the image. See Lagrange's programmatic declaration at the beginning of his *Mécanique analytique* [Lagrange 1787, 2]: «In this work you will not find any image [point de Figures]» (my translation).

²⁸ See *Rep.* 510d5-6: «you also know that they use visible forms and make their arguments about them [τοῖς ὁρωμένοις εἶδεσι προσχρῶνται καὶ τοὺς λόγους περὶ αὐτῶν ποιοῦνται]».

²⁹ In *Eucl.* 201.18-202.1; Proclus 1992, 158 (modified translation).

Understanding the nature of a thing in mathematics is a twofold process. First, we have the λόγος of the entity discovered by the soul's δίανοια, and then its representation as a figurative entity in the φαντασία. If the first kind of understanding ensures the full comprehension of what we are operating with, the second enables us to manipulate geometrical entities. In other words, Proclus articulates in two different faculties what in Plato's *Republic* was included in the δίανοια alone. In his account, the δίανοια refers just to the level of the logical discourse and language, whereas the production of the image is deputized to the activity of the imagination. In doing so, Proclus is systematizing the rare passages where Aristotle shows the crucial role played by the imagination in focusing on the geometrical object.³⁰ However, according to Aristotle the imagination lets the soul detach the figure from the other sensible qualities of the physical substance, while in Proclus the imagination acts by following the λόγος developed by the δίανοια with no relation to the images derived from the senses.³¹

The dynamic between discursive reasoning and imagination is pictured well in the mutual intertwining between theorem and problem:

But again it is impossible to say anything about the construction [συστάσεως] of the parallelograms, or about their equality, without the theory of parallel lines. [...] Hence of necessity he [Euclid] begins his instruction with parallel lines and, after proceeding the short way, turns from them to theory of parallelograms, using as a connecting link between these two portions of the *Elements* a theorem that seems to be examining a property of parallel lines but in fact furnishes the primary genesis [γένεσιν δὲ πρώτη] of the parallelogram.³²

The theorem sets off the property of an entity, which becomes the main parameter for the creation of an image outlined according to that property. Thus, the λόγος in discursive reasoning determines the boundaries of the

³⁰ Of course, this does not mean that Proclus reads Aristotle without any further mediation, cf. Blumenthal 1977 and Mueller 1990. In general, on the background of Proclus' philosophy of mathematics, see Mueller 1987a and O'Meara 1991.

³¹ Cf. Saget 1971 and Nikulin 2010.

³² In Eucl. 355, 1-13; Proclus 1992, 276.

activity of the imagination. In this case, figuring a parallelogram means shaping a figure, thanks to parallel lines with that precise property. Their coming into being is a matter of “projectioning” (προβάλλειν, the same stem from which πρόβλημα derives) the λόγοι onto the imagination.³³ What is important to underline here is that the image refers directly to the discursive definition and *just indirectly to the form* present in the νοῦς. That is, if in Plato’s account the function of the διάνοια is that of *making explicit* that the sensible image is referred to a noetic entity, in Proclus the discursive reasoning represents the *ontological mediation* between the form and the image. Another interesting consequence is that the role played by the διάνοια in Proclus complicates the copy-model metaphor usually taken as the basic Platonic explanation of the relation between the sensible and the noetic realm. Actually, the image is figured out *according to the definition*, i.e. observing the noetic features explicated in it. That means precisely that the λόγος and the image do not reflect each other. Their mutual relationship is not that of a copy to its model, but that of two different representations of the same entity. Their unity and their capacity of a mutual reference is grounded in their common relation (more mediated or not) to the same form. Thus, Proclus can include in the same ontological hierarchy the irremovable inner duality of ancient geometry.

3. *The Art of the Geometrician*

Speaking about the εἶδη looked by the intelligence, Proclus names them the «immutable and eternal forms».³⁴ It is a remarkable coincidence that Descartes makes use of a similar expression in the Fifth Meditation. Referring to the mathematical object the mind finds in its imagination, he points out that «there is still a determinate nature [*quaedam ejus natura*], or essence, or form of [it] which is immutable and eternal».³⁵ In order to understand what these traditional Scholastic concepts could

³³ See Mueller 1987b.

³⁴ In Eucl. 13, 25-26; Proclus 1992, 12 (modified translation).

³⁵ AT VII, 64, 15-16; Descartes 1995, 45.

mean in the context of Cartesian metaphysics,³⁶ it is crucial to consider Descartes' mathematical work. Actually, the entire Second Book of the *Geometry* is explicitly devoted to an illustration of the *nature* of the curved lines. Therefore, it is one of the best candidates for understanding Descartes' strange expressions in the Fifth Meditation.³⁷

Before looking specifically at the curves, we have to sketch briefly the outline of Cartesian geometry. From the beginning, Descartes makes clear the distance of his approach from that of the Ancients: «Any problem in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction».³⁸ First, a detail: a text, whose title is *The Geometry*, does not present any reference to the theorem as one of its main procedures of inquiry and no justification is given for its absence. Descartes just leaves it aside in order to offer to the problem the throne of the reign of geometry. The break with the Ancients could not be clearer.

This means that Cartesian geometry is a unitary field without gaps in itself. Its unity is not given by the ontological stability of the entity it deals with, but by its “being-a-problem” for the geometrician. However, a problem is an epistemic situation whose method of solution can be found by the mind. Thus, “being-a-problem” and “being-solvable” thanks to a set of rules are the same.³⁹ A Cartesian geometrical object is not a self-standing entity, but the outcome of a process of discovering the solution to a given situation. In this sense, it is an “object” as far as it is the solution to a problem. Otherwise, it can be used as a tool in order to solve other problems becoming part of the instruments which the knowing mind can make use of. Of course, this implies that what in Proclus' perspective was a category thanks to which a partial aspect of the practice of the geometrician could be classified in Descartes' eyes becomes the constitutive feature of the *purae Matheseos objectum*.⁴⁰

³⁶ A good analysis of the text of the Fifth Meditation can be found in Doubouclez 2019.

³⁷ As far as I know, the only attempt to read the Fifth Meditation thanks to the *Geometry* is Lachterman 1986.

³⁸ AT VI, 369, 4-7; Descartes 1954, 2.

³⁹ On the notion of problem in Descartes' *Rules* see the Appendix to Ciffoletti 1992.

⁴⁰ For this expression see AT VII, 74, 2.

Descartes' attempt at unifying the realm of geometry grounds one of the most astonishing passages in his *Geometry*, namely his distinction between geometrical and mechanical curves.⁴¹ This distinction is usually considered from the point of view of the movement of the mind. Geometrical curves are those which «can be conceived of as described by a continuous motion or by several successive motions»,⁴² while those which «must be conceived of as described [*on les imagine décrites*] by two separate movements whose relation does not admit of exact determination»⁴³ are the mechanical ones. Nevertheless, the movement is not the basic phenomenon for the distinction. The capacity of figural imagination to make visible the curves with a unique movement is the fruit of the power of the mind to build a method including all the relevant parameters for the construction of the curve (i.e. for the solution of the problem). That means that the construction of the curve must always be the figural production of the setting up of an *equation*. More clearly, the *nature* of a curve is determined by its relation to its method of production, i.e. to its equation. Thus, a curve, which cannot be traced back to an equation or to a set of mutual related equations, is just a fictional entity and therefore it is not part of the geometrical field. In doing so, Descartes is not distinguishing between two different objects in geometry. He is separating in its historical tradition what was truly geometrical and what was not. In other words, the Second Book of the *Geometry* is devoted to distinguishing in the geometrical field what is the fruit of a truthful aggregation from what is a barely fiction of the mind. Its similarity with the question faced in the Fifth Meditation is evident.

The notion of equation is crucial in Descartes' geometrical calculus. It is obtained by relating the elements of a problem to two common measures (the axes) and creating a superior relation between these two. Thus, the previous unrelated and unknown elements are solved into ratios and proportions among knowable parameters. It is important to note that the equation is *a translation of the initial*

⁴¹ The classic interpretation of this distinction can be found in Vuillemin 1960. For a different approach see Chiaravalli 2020.

⁴² AT VI, 390, 1-3; Descartes 1954, 43.

⁴³ AT VI, 390, 13-15; Descartes 1954, 44.

conditions of the problem into a common measure. In other words, it is the problem in itself, yet exposed in a way according to which the elements (which should be fully calculated in order to obtain a complete solution) are designated as such (what we usually call the “unknowns” of an equation). Thus, the equation points to all the geometrical entities defined by those parameters, and the mind is aware of the possibility of completely calculating the outcomes. Its full calculation is a matter of circumstances. The passage from the pure possibility of the equation to the actuality of the outcomes is just the result of a further specification of the invariant represented by the relation among known and unknown terms. What represents the fixed structure of the Cartesian mathematical object is the relational elements among its parts. Therefore, the form or essence of the Fifth Meditation finds its first reference in the formula of the equation. To put it more clearly, in the equation we can find the first Cartesian attempt to reform the notion of generality, which was traditionally represented by the concepts of essence or form.

Of course, this does not mean that the equation *is* the essence. Actually, we have to resist the temptation to consider the equation as the general element, whereas the properly geometrical construction would be its singular representation.⁴⁴ We have to distinguish carefully the pictorial role of the actual geometrical entities in their reference to the algebraic symbols. The singular instantiation of an equation is its individual result. This means that the generality-individuality relationship exists only in the relation between the equation and the *point* of the curve which it describes if completely calculated. Unlike the point, the curve is not the representation of *a* solution to the problem, but the image of *all the possible solutions to that kind of problems*. If this is true, then the curve is *a representation of the equation as such*. In other words, if the point is the fruit of a full explication of the values of the algebraic formula, the curve is not. On the contrary, it is an attempt to geometrically represent *the features of generality and pure possibility of the algebraic complex*. This implies a deep change in the understanding of the capacity of the imagination.

Recall that in Proclus the mathematical imagination is completely

⁴⁴ In my opinion, this is the basic mistake in the otherwise amazing Lachterman 1986.

independent of the senses and its objects are the fruit of its interaction with the discursive reasoning. In this way, the imaginative capacity is the bearer of an epistemic logic which has its own legitimacy, not derivable from that of the senses. Apparently, the same argument is present in Descartes' *Rules for the Direction of the Mind* where we are told that, when the intellect applies itself «to the imagination in order to form new figures, it is said to imagine or conceive».⁴⁵ Without entering in such a difficult topic as that of the role of the imagination in the *Rules*,⁴⁶ what is important to remember is that an understanding of the productive capacity of the imagination was present even in the Scholastic thought and in the debate on the nature of mathematics in 16th century.⁴⁷ Consequently, the actual innovation in the modern framework is not the productivity of the imagination as such, but *what it produces*. As our brief summary of Descartes' method in the *Geometry* has shown, the crucial point is that the figural geometrical object is a representation of the generality of the algebraic formula. The Cartesian image is not an individual which needs a general entity, which it can refer to, in order to obtain legitimacy and intelligibility. It is not a barely imaginative exemplar, but a figural archetype. Kant is on the same line when he writes in the *Critique of Pure Reason* that the «representation of a general procedure of the imagination for providing a concept with its image is [...] the schema for this concept».⁴⁸ The true specific feature of Modern imagination is that it can produce a generality and not its bare example. Consequently, the great challenge Descartes and his heirs have to face is how to distinguish the generality of the imagination from that of the intellect.

An easy solution to this extremely difficult question would be to say that the equation is the representative of the intellect in the geometric realm. Thus, the distinction between equation and curve would be the same as that between intellect and imagination. Unfortunately, this is not

⁴⁵ AT X, 416, 2-4; Descartes 1995, 42.

⁴⁶ See on this point at least Marion 1975, Pasini 1992, and Sepper 1996.

⁴⁷ On this point Rabouin 2009 is absolutely right.

⁴⁸ *KrV* A140/B179-180; Kant 1998, 273. On this see Klein 1985. Of course, this does not mean that Descartes' and Kant's understanding of imagination is the same, see Ferrarin 1995.

the case. In the *Rules* Descartes is extremely clear in stating that even the algebraic symbols are products of the imagination.⁴⁹ The equation is part of the generality of the imagination, just like the curve. The passage from one to the other is a matter of different representations, without any change in the faculty involved. Using Leibniz's terms, the curve is the *expression* of the equation and reverse.⁵⁰

This terminology fits particularly well our puzzling situation because it prevents a possible misunderstanding. Actually, the argument of the Fifth Meditation has been often read as if Descartes would describe the discovery of a separate essence looked through the image of the geometrical object. Now, if this is certainly true of the idea of God, the same does not apply to the other ideas. In order to note their difference, we have to appreciate a textual detail. We saw that concerning the geometrical objects Descartes speaks of *quaedam ejus natura, sive essentia, sive forma, immutabilis et aeterna*. On the contrary, in the case of God's idea the formulation is a bit different: «There are many ways in which I understand that this idea is not something fictitious which is dependent on my thought, but is an image of a true and immutable nature [*sed imaginem verae et immutabilis naturae*]». ⁵¹ This last expression is used in order to underline the preeminence of God's idea among the other innate ideas and this same pre-eminence is grounded in the necessary connection between God's essence and his existence. Thus, a separation of the immutable and eternal essence from its image

⁴⁹ Cf. AT X, 416.28-417. 15; Descartes 1985, 43: «If, however, the intellect proposes to examine which can be referred to the body, the idea of that thing must be formed as distinctly as possible in the imagination. In order to do this properly, the thing itself which this idea is to represent should be displayed to the external senses. [...] it is not the things themselves which should be displayed to the external senses, but rather certain abbreviated representations of them [...]». This passage from Rule XII finds its accomplishment with the *brevissimas notas* (i.e. the algebraic symbols) in Rule XVI, cf. AT X, 455, 4. On this point see Caton 1973.

⁵⁰ See the famous definition in Leibniz's *Quid sit idea* [Leibniz 1890, VII, 263]: «*Exprimere aliquam rem dicitur illud, in quo habentur habitudines, quae habitudinibus rei exprimendae respondent*». Not surprisingly, one of Leibniz's examples is that «*aequatio Algebraica exprimit circulum aliamve figuram*». As far as I know, the best analysis of the notion of expression in Leibniz is Mugnai 1976.

⁵¹ AT VII, 68, 12; Descartes 1995, 47.

in the mind is *a privilege of the divine idea*. Otherwise, the essence is the very structure of the image in itself and coincides with the fact that *demonstrari possint variae proprietates* of the figure. In other words, the essence of a geometrical object coincides with the operational possibilities which the mind has in dealing with it.⁵² As we saw in the case of the equation, in Descartes' understanding, a mathematical object is nothing other than its being a tool of the knowing activity of the subject. In this context, an essence points to the set of actions which the mind can perform.

Coming back to the much more complex account of the *Geometry*, we could say that the essence is precisely the fact that the mind can operate on the equation or on the curve knowing that their properties are the same. Making evident this correspondence is one of Descartes' main preoccupations. From the point of view of the bare algebraic calculation, Cartesian *Geometry* does not provide any relevant innovation. Algebra is, in a certain way, given.⁵³ Technically speaking, Descartes' goal is that of creating a geometrical semantics for the algebraic reckoning.⁵⁴ From the point of view of his epistemology, this means that the task of the *Geometry* is to show that one can operate on the equation without losing information on the curve and vice versa.⁵⁵ This means that the correspondence of the operational properties between the curve and the equation is what makes of the one the expression (again in Leibnizian

⁵² On mathematics as the domain of possibility see the *Conversation with Burman* in AT V, 160. Kant shows to be a loyal Cartesian when he writes in the *Metaphysical Foundations of the Natural Science* that [Kant 2004, 3]: «Essence is the first inner principle of all that belongs to the possibility of a thing. Therefore, one can attribute only an essence to geometrical figures, but not a nature (since in their concept nothing is thought that would express an existence)».

⁵³ Of course, this does not mean that Descartes does not include any change at all, especially from the point of view of the symbolic notations. On this point see Serfati 2005.

⁵⁴ See Chiaravalli 2020.

⁵⁵ This is precisely the result of the “doctrine of the equations” in the Third Book. Remind that in Descartes the equation has no legitimacy outside its geometrical problem. It does not exist as autonomous object of consideration as it does in Fermat, for example. On the relation between algebraic formula and geometrical construction see Bos 2003.

terms) of the other. They have the same essence because they are two different representations of the same operational context.

In this way, the unitary and closed realm of geometry is completely ensured. The key change lies in the will of the mind to represent an object according to the peculiar problem it has to solve.⁵⁶ Cartesian geometry is an artful use of our figural and symbolic imagination. Its power is crucial in Descartes' mathematical account. It makes visible those portions of space which should be included in the solution to the problem. Without its help no clear mathematical operation would be possible. Actually, the geometrical objects usually employed by Descartes are quite different from the triangle, which is used as example in the Fifth Meditation in compliance with the traditional Scholastic debate on the nature of mathematical demonstration.⁵⁷ There are no simple, individual figures in Descartes' *Geometry*, but different kinds of curves which isolate specific parts of the extension. Thus, imagination has the crucial task to identify all the components of the portion of space which the mind is interested in. Following the intention of the intellect, it detaches that specific part from its original extensional context, letting it appear as an independent object. Thus, that part of the extension, now conceived as one and singular object, shows a specific set of operational properties. In this way, it appears as a peculiar essence despite the fact that Descartes abolishes all the substantial forms from the material world. In fact, it has or, better, it *is* an essence only in so far as it is not a part of the physical universe, but the isolated portion of the clear idea of extension which the intellect finds in the imagination.⁵⁸

⁵⁶ On the will as basic dimension of the Cartesian mind see the *Passions of the Soul*, AT XI, 339, 7-8; Descartes 1985, 333: «our volition, which is the only, or at least the principal, activity of the soul».

⁵⁷ See on this Bernhardt 1988.

⁵⁸ See the Sixth Meditation, AT VII, 73, 25-26: «*ea naturae corporeae idea distincta, quam in imaginatione mea invenio*».

4. Conclusion: Recasting Platonism in Modernity

A conclusion we can draw from our brief sketch of Proclus' and Descartes' understanding of mathematics is that their distance is irreconcilable. This is because what they are speaking about when they reflect on geometry are two completely different practices. Reading Descartes as a Platonist means to overlook the fact that modern intellectuals cannot resist the temptation to adapt the ancient conceptuality to the phenomena of their new mathematics and culture in general (even when they want to be loyal followers of the Ancients). It goes without saying that this adaptation implies a radical betrayal of the ancient heritage. Thus, our principal task should not be to take for granted metahistorical categories, such as "Platonism", but to carefully detect the breaks in the apparent continuity in the tradition of philosophy (and mathematics, in this case). From this point of view, the historical problem of Platonic legacy in modernity is immediately complicated into three different aspects: the radical difference among the phenomena taken into account (in our case, mathematics, but the same might be said about physics or society), the transformation of the contents of the dialogues and of the writings of Plato's Ancient followers according to the new phenomena, and the self-awareness of the betrayal necessarily implicit in this operation. One of the trickiest problems in the history of "Platonism" is that this self-awareness is usually not present in the authors as well as in their interpreters. Thus, references to the Platonic texts are taken as if they were evidence of an engagement in a trustful continuation of Plato's work (assuming that we are able to properly understand what Plato's work was about).

Beyond the common stereotype created by this misunderstanding, the real question concerns the different ways in which modern authors make use of ancient concepts and, in our context, of those belonging to the Platonic tradition. Put in this way, the true problem concerning Descartes' Fifth Meditation is: why does he use a Platonic language in order to express something radically different from the usual meaning of that language? A full and satisfying answer cannot be given here. A modest proposal could be that of considering the huge role which the rhetorical strategy plays in Cartesian texts, with particular regard to the *Meditations*, which is a masterful and playful composition of different and contrastive languages. If one of the main goals of this literary *pastiche*

is that of creating the impression of a comfortable and peaceful relation with the previous tradition, this cannot explain all the rhetoric strategies Descartes makes use of in the text. In the Fifth Meditation, for instance, he employs the language of Platonism in order to distinguish *inside the mind* two different levels of thinking and, in doing so, grounding the capacity of the human thought to operate truthfully without any reference to external entities. In this way, the historical meaning of traditional categories is completely transvaluated, but the language of pre-Cartesian metaphysics is saved for the theory of knowledge (as the survival of the couple “matter-form” in the post-Cartesian debate shows at best). Essence as a descriptive category of the structure of the individual object is something that is possible only in the domain of thinking. Descartes uses it in order to point to the *mind’s awareness of achieving truth while operating with that idea*. With all the irony of which history is capable, the concept of essence is used precisely so that it can be abolished.

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Keywords

Descartes; Proclus; Platonism; geometry; imagination

Abstract

The aim of this article is to question a common interpretation of Cartesian philosophy of mathematics according to which Descartes is a Platonist. Such a controversial issue is faced by contrasting Cartesian geometry with Proclus' commentary to Book I of Euclid's *Elements*, which in the 16th century was the main source for a Platonic interpretation of mathematics. Despite many apparently common aspects and concepts, we will see that Proclus' and Descartes' accounts are mutually irreconcilable. This is the case because the kinds of mathematics, which they are trying to philosophically explain, are completely different. Actually, Euclidean geometry is structurally based on two irreducible elements: the image and the word. Thus, Proclus is forced to articulate an epistemology that is able to account for a mathematical practice which is intrinsically divided, creating a hierarchy among its elements. On the contrary, Cartesian geometrical calculus is a unitary field where the language of proportions includes the inner duality of ancient geometry. From this basic difference we can show how distant they are on the role of the epistemic faculties in mathematics and especially on that of the imagination. This makes us possible to look differently at Descartes' usage of Platonic conceptuality in the Fifth Meditation. Concepts like essence or form are no longer notions corresponding to different levels of being, but rather representatives of the different operations of the human mind in producing its own instruments of knowledge.

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